

Q3: Deduce the gas laws from kinetic gas equation?

Ans:  $\rightarrow$  Boyle's law:

From the kinetic gas equation we know,

$$PV = \frac{1}{3} m n u^2$$

If total mass of the gas is  $M$   
then,  $M = m \cdot n$

$$\therefore PV = \frac{1}{3} M u^2$$

$$\propto PV = \frac{2}{3} \times \frac{1}{2} M u^2$$

$$\propto PV = \frac{2}{3} [K.E.] \quad \left[ \frac{1}{2} M u^2 = \text{Kinetic energy} \right]$$

According to postulate kinetic energy is directly proportional to absolute temperature. Hence, at constant temperature kinetic energy is constant.

$\therefore PV = \text{Constant}$ ; when temperature is constant.

This is Boyle's law.

Charles's law:  $\rightarrow$

From above equation

$$PV = \frac{2}{3} [K.E.]$$

$$\propto V = \frac{2}{3} \cdot \frac{1}{P} [K.E.]$$

If  $P$  is constant we may write

$$V = \text{constant} \times [K.E.]$$

$$\therefore V \propto [K.E.]$$



But according to postulate

$$[K.E.] \propto T$$

Hence,  $V \propto T$ ; When Pressure is constant

This is Charles's law.

### Avogadro's Hypothesis:

Let us consider two gases. Gas no. 1 and gas no. 2. Using suffixes 1 and 2 respectively for the gases the kinetic gas equation may be written as,

$$P_1 V_1 = \frac{1}{3} m_1 n_1 u_1^2 \quad \text{--- (i)}$$

$$P_2 V_2 = \frac{1}{3} m_2 n_2 u_2^2 \quad \text{--- (ii)}$$

Equal volume of two gases at the same temperature and pressure are considered

Hence  $P_1 V_1 = P_2 V_2$

Thus  $\frac{1}{3} m_1 n_1 u_1^2 = \frac{1}{3} m_2 n_2 u_2^2$

or,  $m_1 n_1 u_1^2 = m_2 n_2 u_2^2 \quad \text{--- (iii)}$

Because both the gases are at equal temperature the kinetic energy per molecule should also be equal.

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_2 u_2^2$$

or,  $m_1 u_1^2 = m_2 u_2^2 \quad \text{--- (iv)}$

Dividing equation (iii) by (iv) we get

$$\frac{m_1 n_1 u_1^2}{m_1 u_1^2} = \frac{m_2 n_2 u_2^2}{m_2 u_2^2}$$

or,  $n_1 = n_2$

Hence equal volume of gases at the same temperature and pressure contain the same number of molecules. This is Avogadro's Hypothesis.



We know that

$$PV = \frac{1}{3} mn u^2$$

The total mass of the gas is  $M$ .

Hence  $M = m \cdot n$ .

$\therefore PV = \frac{1}{3} M u^2$

or  $u^2 = \frac{3PV}{M}$  [But  $\frac{M}{V} = D$  density]

or  $u^2 = 3P \frac{1}{D}$

or  $u = \sqrt{3P \frac{1}{D}}$

If pressure is constant.

$$u = \frac{\text{Constant}}{\sqrt{D}}$$

$\therefore u \propto \frac{1}{\sqrt{D}}$

It is obvious that rate of diffusion  $r$  is directly proportional to  $u$ .

$$r \propto u$$

Hence  $r \propto \frac{1}{\sqrt{D}}$

Hence, rate of diffusion of a gas is inversely proportional to the square root of its density. This is Graham's law of diffusion.

